

1. Details of Module and its structure

Module Detail	
Subject Name	Physics
Course Name	Physics 02 (Physics Part-2, Class XI)
Module Name/Title	Unit 7, Module 7, Viscosity Chapter 10, Mechanical Properties of Fluids
Module Id	Keph_201002_eContent
Pre-requisites	Elasticity, fluid properties, stress and strain, pressure in fluids.
Objectives	<p>After going through this lesson, the learners will be able to:</p> <ul style="list-style-type: none"> • Understand fluid friction - viscosity • Describe the cause of viscosity. • Explain the factors affecting viscosity • Derive an expression for coefficient of viscosity. • Deduce Stokes' law • Conceptualize terminal velocity. • Derive the expression for the terminal velocity
Keywords	Viscosity, stokes' law, terminal velocity, coefficient of viscosity, unit of viscosity, body falling in a viscous fluid , viscous drag

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1. UNIT SYLLABUS

UNIT 7:

PROPERTIES OF BULK MATTER:

24 periods

Chapter–9: Mechanical Properties of Solids:

Elastic behaviour, Stress-strain relationship, Hooke's law, Young's modulus, bulk modulus, shear, modulus of rigidity, Poisson's ratio, elastic energy.

Chapter–10: Mechanical Properties of Fluids:

Pressure due to a fluid column; Pascal's law and its applications (hydraulic lift and hydraulic brakes). Effect of gravity on fluid pressure. Viscosity, Stokes' law, terminal velocity, streamline and turbulent flow, critical velocity, Bernoulli's theorem and its applications. Surface energy and surface tension, angle of contact, excess of pressure across a curved surface, application of surface tension ideas to drops, bubbles and capillary rise

Chapter–11: Thermal Properties of Matter:

Heat, temperature, thermal expansion; thermal expansion of solids, liquids and gases, anomalous expansion of water; specific heat capacity; C_p , C_v - calorimetry; change of state - latent heat capacity. Heat transfer-conduction, convection and radiation, thermal conductivity, qualitative ideas of Blackbody radiation, Wien's displacement Law, Stefan's law, Greenhouse effect.

2. MODULE-WISE DISTRIBUTION OF UNIT SYLLABUS**17 MODULES**

Module 1	<ul style="list-style-type: none"> • Forces between atoms and molecules making up the bulk matter • Reasons to believe that intermolecular and interatomic forces exist • Overview of unit • State of matter • Study of a few selected properties of matter • Study of elastic behaviour of solids • Stationary fluid property: pressure and viscosity • Stationary liquid property: surface tension • Properties of Flowing fluids • Effect of heat on matter
Module 2	<ul style="list-style-type: none"> • Idea of deformation by external force • Elastic nature of materials • Elastic behaviour • Plastic behaviour • Tensile stress • Longitudinal Stress and longitudinal strain • Relation between stress and strain • Hooke's law • Young's modulus of elasticity 'Y'
Module 3	<ul style="list-style-type: none"> • Searle's apparatus • Experiment to determine Young's modulus of the material of a wire in the laboratory • What do we learn from the experiment?
Module 4	<ul style="list-style-type: none"> • Volumetric strain • Volumetric stress • Hydraulic stress • Bulk modulus K • Fish, aquatic life on seabed, deep sea diver suits and submarines
Module 5	<ul style="list-style-type: none"> • Shear strain • Shear stress • Modulus of Rigidity G

	<ul style="list-style-type: none"> ● Poisson's ratio ● Elastic energy ● To study the effect of load on depression of a suitably clamped meter scale loaded at i)its ends ii)in the middle ● Height of sand heaps , height of mountains
Module 6	<ul style="list-style-type: none"> ● Fluids-liquids and gases ● Stationary and flowing fluids ● Pressure due to a fluid column ● Pressure exerted by solid , liquids and gases ● Direction of Pressure exerted by solids, liquids and gases
Module 7	<ul style="list-style-type: none"> ● Viscosity- coefficient of viscosity ● Stokes' Law ● Terminal velocity ● Examples ● Determine the coefficient of viscosity of a given viscous liquid by measuring terminal velocity of a given spherical body in the laboratory
Module 8	<ul style="list-style-type: none"> ● Streamline and turbulent flow ● Critical velocity ● Reynolds number ● Obtaining the Reynolds number formula using method of dimensions ● Need for Reynolds number and factors effecting its value ● Equation of continuity for fluid flow ● Examples
Module 9	<ul style="list-style-type: none"> ● Bernoulli's theorem ● To observe the decrease in pressure with increase in velocity of a fluid ● Magnus effect ● Applications of Bernoulli's theorem ● Examples ● Doppler test for blockage in arteries
Module 10	<ul style="list-style-type: none"> ● Liquid surface ● Surface energy ● Surface tension defined through force and through energy ● Angle of contact ● Measuring surface tension

<p>Module 11</p>	<ul style="list-style-type: none"> ● Effects of surface tension in daily life ● Excess pressure across a curved liquid surface ● Application of surface tension to drops, bubbles ● Capillarity ● Determination of surface tension of water by capillary rise method in the laboratory ● To study the effect of detergent on surface tension of water through observations on capillary rise.
<p>Module 12</p>	<ul style="list-style-type: none"> ● Thermal properties of matter ● Heat ● Temperature ● Thermometers
<p>Module 13</p>	<ul style="list-style-type: none"> ● Thermal expansion ● To observe and explain the effect of heating on a bi-metallic strip ● Practical applications of bimetallic strips ● Expansion of solids, liquids and gases ● To note the change in the level of liquid in a container on heating and to interpret the results ● Anomalous expansion of water
<p>Module 14</p>	<ul style="list-style-type: none"> ● Rise in temperature ● Heat capacity of a body ● Specific heat capacity of a material ● Calorimetry ● To determine specific heat capacity of a given solid material by the method of mixtures ● Heat capacities of a gas have a large range ● Specific heat at constant volume C_V ● Specific heat capacity at constant pressure C_P
<p>Module 15</p>	<ul style="list-style-type: none"> ● Change of state ● To observe change of state and plot a cooling curve for molten wax. ● Melting point, Regelation, Evaporation, boiling point, sublimation ● Triple point of water ● Latent heat of fusion ● Latent heat of vaporisation ● Calorimetry and determination of specific latent heat

	capacity
Module 16	<ul style="list-style-type: none"> • Heat Transfer • Conduction, convection, radiation • Coefficient of thermal conductivity • Convection
Module 17	<ul style="list-style-type: none"> • Black body • Black body radiation • Wien's displacement law • Stefan's law • Newton's law of cooling, • To study the temperature, time relation for a hot body by plotting its cooling curve • To study the factors affecting the rate of loss of heat of a liquid • Greenhouse effect

MODULE 7

3. WORDS YOU MUST KNOW

Fluids: A fluid is a substance that can flow. This includes both gases and liquids.

Stress: The restoring force per unit area is known as stress. The restoring force is equal to the applied force. If F is the force applied and A is the area of cross section of the body, Magnitude of the stress = F/A . The SI unit of stress is N m^{-2} or pascal (Pa)

Shearing strain: Shearing strain is defined as the ratio of relative displacement of the faces Δx of a cylinder to the length L .

$$\text{Shearing strain } \frac{\Delta x}{L} = \tan \theta$$

4. INTRODUCTION

You must have experienced that whenever you try to apply butter taken out directly from refrigerator on your toast you face some difficulty. But when butter is kept outside for some time it becomes easy to apply. Have you ever thought of your favorite honey as a liquid? If you pour honey in some other container it will slowly slide or flow downward. But If you pour water, it will flow down very quickly. What makes this happen?

Can you imagine trying to swim in a pool of honey instead of water?



Fig: honey is actually a liquid. It flows very slowly

We know gravity pulls everything with the same acceleration. So objects of same mass experience same gravitational force. **In fluids even when their mass is the same the rate of fall is different**

Butter, honey, motor oil and water. flow at different speeds. Why is that?

Well, the answer is that each of these has a different viscosity. **Viscosity** is the property of a liquid that describes how fast or slowly it will flow. A liquid with larger viscosity - that is thick, like honey will flow slowly. A liquid with low viscosity, or a thin liquid, like water, will flow quickly.

Viscosity is the measure of resistance to flow which arises due to the internal friction between the layers of fluid as they slip past one another when fluid flows

In this module we will learn about the concept of **viscosity** and its dependence on temperature. We will also discuss the use of knowledge of viscosity in our day to day life.

5. VISCOSITY

Most of the fluids offer some resistance to motion. This resistance to fluid motion is like an internal friction, analogous to friction which comes into play when a solid move on a surface.

The property of a fluid by virtue of which an internal frictional force acts between its different layers and opposes their relative motion is called viscosity.

The backward dragging force, called viscous force, acts tangentially on the layers of the fluid in motion.

Intermolecular Forces Affect the Viscosity of a Substance

The viscosity of a substance is related to the strength of the intermolecular forces acting between its molecular units. In the case of water, these forces are primarily due to hydrogen bonding. Liquids such as syrups and honey are much more viscous because the sugars they contain are studded with hydroxyl groups ($-OH$). They can form multiple hydrogen bonds with water and with each other, producing a sticky disordered network which makes flow much more difficult (resulting in high viscosity).

In much of our daily experience, we act as viscometers, sensing the viscosity of liquids and assessing their fitness for a particular purpose.

For example, some people add cream and sugar to coffee or tea not only for the taste but for what they term the mouth feel.

In this case, we are forcing the fluid through a small gap between our tongue and palate, sensing the thickness and smoothness. Cream and sugar add particulate solids to the watery tea and coffee, which add to the viscosity of the solution.

In another setting, we judge the quality of a sauce by the way it flows and adheres to certain foods - salad dressing to lettuce, jam or jelly to toast, ketchup or mayonnaise to fried foods - the flow and cohesion are related to the viscosity. If you were to place honey or corn syrup in the refrigerator, it thickens considerably relative to its thickness at room temperature. We can see that viscosity is highly dependent on temperature.



Water



Honey

In everyday terms, viscosity is described as thickness or internal friction within the substance. Therefore, we say water is *thin*, having a low viscosity, while honey is *thick*, having a high viscosity. When a fluid is less viscous, it flows more easily. That is why it is easy to pour water instead of honey into the other container.

Let us try to find viscosity of different liquids through an experiment. We will name our experiment as

METAL PELLET (cycle ball bearing) RACE: TO FIND VISCOSITY OF VARIOUS FLUIDS.

To perform this experiment, we need:



Watch video for help with the experiment

Tall graduated cylinder,

Metal pellet, Stopwatch, Cooking oil, jam, Shower gel, Honey, Glycerin, Liquid glue, Hand sanitizer. Water and any other liquid you want to test.

Procedure:

1. Fill the graduated cylinder with one of the sample liquids. Leave some gap at the top so it does not overflow. Fill cylinder up to the same height each time.
2. Hold the pellet at the opening of the graduated cylinder in one hand and the stop watch in the other hand.
3. Drop the pellet and simultaneously start the stopwatch.
4. Stop the timer when the pellet touches the bottom of the cylinder.
5. Record the name of the liquid you tested, the original height of the liquid, and how long it took for the pellet to fall in seconds.
6. Repeat the step 2 more times for each liquid

Calculate the average time.

$$\text{Average time} = (\text{time 1} + \text{time 2} + \text{time 3}) / 3$$

7. Calculate the average velocity of the pellet through the liquid. The distance is the height of the liquid, and the time will be the average time calculated in Step 6.

$$\text{velocity} = \text{distance} / \text{time}$$

8. Repeat the experiment testing other liquids.

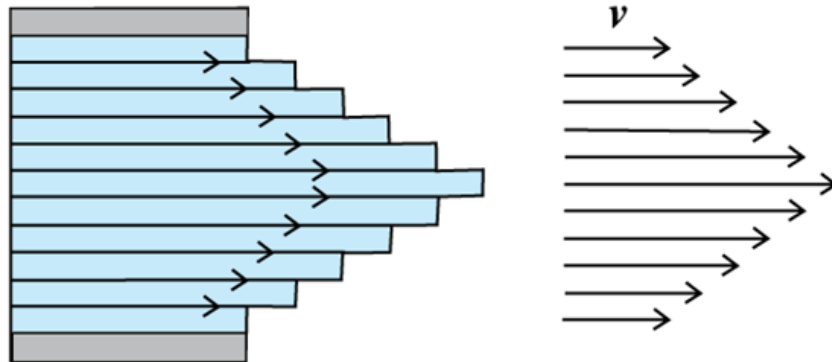
How can you tell which liquids are thicker and which are thinner using the velocity?

To compare the relative viscosities of liquids, it is easy to use the calculated velocities. **Liquids in which the pellet had the slowest velocity had the highest viscosity.** Friction between the molecules of a fluid resists fluid change and deformation. The weight of the pellet, which is the gravitational force, also causes stress on the liquid. High viscosity fluids like honey and glue resist changes caused by these forces the best.

Viscosity of liquids is often very temperature sensitive, with most liquids and gases becoming less viscous (thinner) as they heat up. You can imagine this with hot glue or melted chocolate. To take this experiment further, you may heat up the liquid for a short period of time to see if the pellet drops through faster when the liquid is warmed.

6. CAUSE OF VISCOSITY

Suppose we consider a fluid, like an oil, enclosed between two glass plates. The bottom plate is fixed while the top plate is moved with a constant velocity v relative to the fixed plate. If oil is replaced by honey, a greater force is required to move the plate with the



velocity distribution for viscous flow in a pipe.

Hence we say that honey is more viscous than oil. The fluid in contact with a surface has the same velocity as that of the surface. Hence, the layer of the liquid in contact with top surface moves with a velocity v and the layer of the liquid in contact with the fixed surface is stationary. The velocities of layers increase uniformly from bottom (zero velocity) to the top

layer (velocity v). For any layer of a liquid, the upper layer pulls it forward while the lower layer pulls it backward. The upper fast-moving layer tends to accelerate the lower slow-moving layer while the slow-moving layer tends to retard the fast-moving layer. This results in a force between the layers which tends to destroy the relative motion.



This video shows that the liquid flows in layers.

https://youtu.be/p08_KITKP50

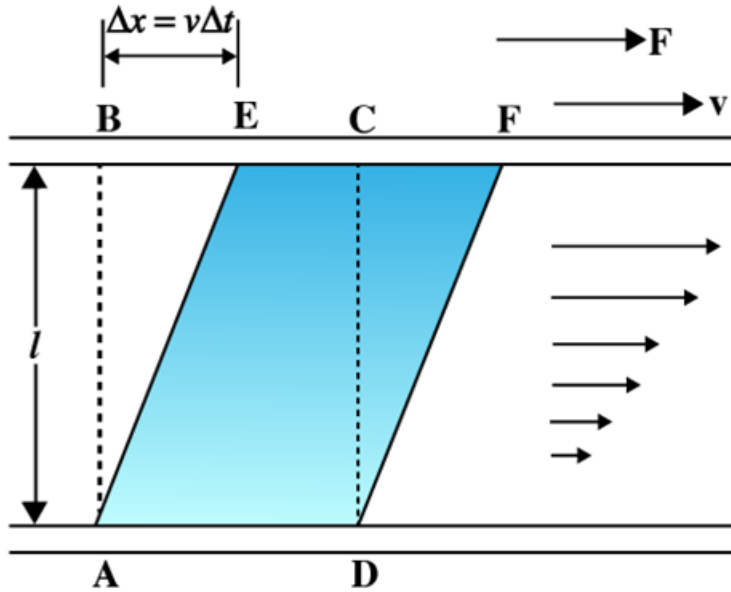
If the slow moving layer tends to retard the fast moving layer, doesn't the fast moving layer tend to accelerate the slow moving layer?



Watch video for basics of viscosity

7. EXPRESSION FOR COEFFICIENT OF VISCOSITY

Suppose we consider a fluid, like an oil, enclosed between two glass plates as shown in Fig.



A layer of liquid sandwiched between two parallel glass plates in which the lower plate is fixed and the upper one is moving to the right with velocity v

On application of force F , a portion of liquid, which at some instant has the shape $ABCD$, takes the shape of $AEFD$ after short interval of time (Δt). During this time interval the liquid has undergone a shear strain of magnitude $\Delta x/l$. The strain in a flowing fluid increases with time continuously.

Unlike solids, in fluids, the stress is found experimentally to depend on ‘rate of change of strain’ or ‘strain rate’ i.e. $\Delta x/(l \Delta t)$ or v/l instead of strain itself.

i.e. Stress \propto strain rate

The coefficient of viscosity (pronounced ‘eta’) for a fluid is defined as the ratio of shearing stress to the strain rate.

$\eta = \text{shearing stress} / \text{strain rate}$

$$\eta = \frac{F/A}{v/l} = \frac{Fl}{vA}$$

Does the coefficient of viscosity for a liquid placed between two solid surfaces depends upon the width separation between solid surfaces?

8. SI UNIT OF VISCOSITY

The SI unit of viscosity is **poiseuille (PI)**. Its other units are N s m^{-2} or Pa s .

The dimensions of **viscosity** are $[ML^{-1}T^{-1}]$.

Generally thin liquids like water, alcohol etc. are less viscous than thick liquids like coal-tar, blood, glycerin etc.



Concrete is mixed with coal tar to make the road layering easy
[http://static.biznes-gazeta.ru/db.biznes-gazeta.ru/images/i\(173\).jpg](http://static.biznes-gazeta.ru/db.biznes-gazeta.ru/images/i(173).jpg)



Glucometer used for checking blood sugar content takes in the blood slowly as the blood is viscous

https://upload.wikimedia.org/wikipedia/en/thumb/2/26/Blood_Glucose_Testing.JPG/280px-Blood_Glucose_Testing.JPG

The coefficients of viscosity for some common fluids are listed in the following Table:

Table: **The viscosities of some fluids**

Fluid	T(°C)	Viscosity (mPl)
Water	20	1.0
	100	0.3
Blood	37	2.7
Machine Oil	16	113
	38	34
Glycerine	20	830
Honey		200
Air	0	0.017
	40	0.019

9. VARIATION OF VISCOSITY WITH TEMPERATURE

- The viscosity of liquids decreases with increase in temperature. This is due to the decrease in cohesion force between the molecules. The cohesion forces decrease due to increase in spacing between molecules with the rise in temperature.
- The viscosity of gases increases with increase in temperatures as $\eta \propto \sqrt{T}$.

Why viscosity of gases shows different behaviour than liquids when we increase the temperature of both?

This is due to the increase in the rate of diffusion of gases from one moving layer to the other because the random motion of atoms increases.

- Experiments with blood show relation between blood flow variation due to body temperature. This implies what happens to the viscosity of blood in the body when the person has fever?

Blood viscosity can become as great as 10 times that of water, and its flow through **blood** vessels is greatly retarded **because of** increased resistance to flow. This will **lead** to decreased oxygen delivery to various parts of the body.

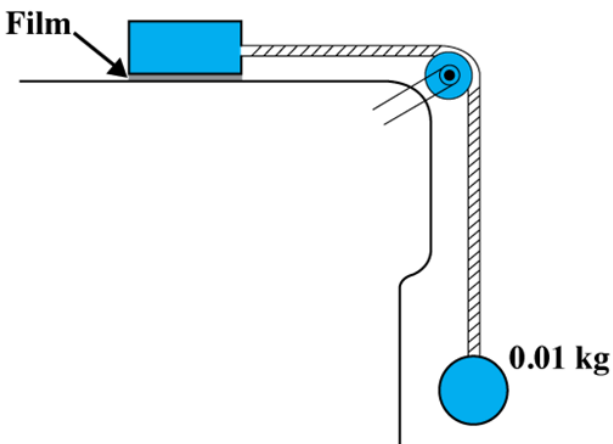
When the blood temperature decreased from 36.5° to 22°C, the mean blood free flow time increased from 11.62 to 15.55 sec (26.13%).

10. APPLICATION OF VISCOSITY

- The knowledge of the coefficient of viscosity of different oils and its variation with temperature helps us to select a suitable lubricant for a given machine in different seasons.
- The knowledge of viscosity of some organic liquids is used in determining the molecular weight and shape of large organic molecular like proteins and cellulose etc.
- Millikan used the knowledge of viscosity in determining the charge on an electron
- The knowledge of Blood viscosity helps doctors to monitor heart, blood pressure related diseases and organ monitoring

EXAMPLE

A metal block of area 0.10 m^2 is connected to a 0.010 kg mass via a string that passes over an ideal pulley (considered massless and frictionless), as in Fig. A liquid with a film thickness of 0.30 mm is placed between the block and the table. When released the block moves to the right with a constant speed of 0.085 m s^{-1} . Find the coefficient of viscosity of the liquid



Measurement of the coefficient of viscosity of a liquid.

SOLUTION

The metal block moves to the right because of the tension in the string. The tension T is equal in magnitude to the weight of the suspended mass m.

Thus the shear force F is

$$F = T = mg = 0.010 \text{ kg} \times 9.8 \text{ m s}^{-2} = 9.8 \times 10^{-2} \text{ N}$$

stress on the fluid = F/A

$$= \frac{9.8 \times 10^{-2} \text{ N}}{0.10 \text{ m}^2}$$

$$\text{Strain rate} = \frac{v}{l} = \frac{0.085 \text{ m/s}}{0.3 \times 10^{-3} \text{ m}}$$

$$\eta = \frac{\text{stress}}{\text{strain rate}}$$

$$= 3.45 \times 10^{-3} \text{ Pa s}$$

THINK ABOUT THESE

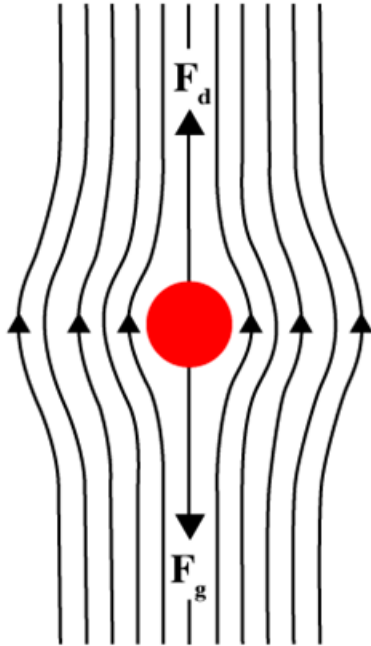
- Does the knowledge of viscosity of different petroleum products help in their excavation from the basin?
- Why does milk in a glass stirred with a spoon comes into rest after some time?
- If two glasses containing oil and milk are stirred vigorously then which of the liquid will come to rest earlier and why?
- A steel-ball, a tennis-ball, a plastic ball of same size and a very small drop of honey are dropped from the roof a high tower. Considering the viscosity and buoyancy of air, which of them will come to the ground first and which will come last?
- Why do clouds appear floating in the sky?

11. STOKES' LAW

This law gives an expression for the viscous force experienced by a body (a spherical) moving through a fluid. This expression was given by an English scientist, **Sir George G. Stokes**.

When a body falls through a fluid it drags the layer of the fluid in contact with it. A relative motion between the different layers of the fluid is set and as a result the body experiences a retarding force.

Force experienced by Falling raindrops and swinging pendulum bob are some common examples of such motion.



https://upload.wikimedia.org/wikipedia/commons/thumb/a/ae/Stokes_sphere.svg/403px-Stokes_sphere.svg.png

The figure shows F_g as weight of the spherical object and F_d as the viscous drag

When a spherical object is dropped in a fluid, it is seen that the viscous force F_d or F experienced by the object is proportional to the

- velocity v of the object through the fluid
- viscosity η of the fluid and
- Radius ' a ' of the sphere.

Thus $F \propto \eta a v$

Here the constant of proportionality is found to be 6π

Thus,

$$F = 6 \pi \eta a v$$

This is known as Stokes' law.

You can obtain the same result in another way

From

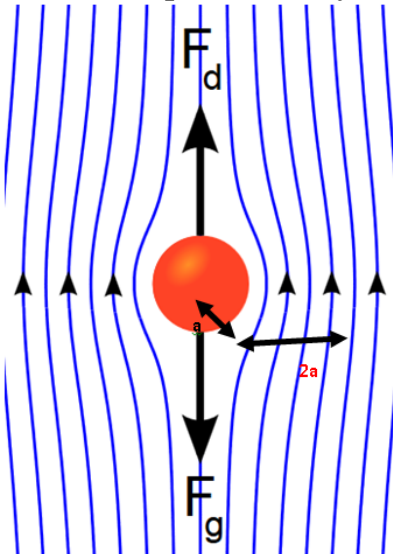
$$\eta = \frac{F/A}{v/l} = \frac{F l}{v A}$$

$$F = \eta v A/l$$

If the velocity of the falling body becomes constant due to net force on it reducing to zero, the viscous force F can be calculated

Using surface area as $4\pi a^2$

$l = a$ as the liquid layers displaced by the spherical body falling in a stationary fluid is = radius of the spherical body.



So the velocity gradient

$$\frac{v}{l} = \frac{v}{a}$$

Or

$$F = \eta 4 \pi a^2 \frac{v}{a} = 4 \pi \eta a v$$

Experiments showed that if the expression is to replace 4 by 6 the value of viscous force is given accurately by the relation

$$F = 6 \pi \eta a v$$

This law is an interesting example of retarding force which is proportional to velocity.

As the velocity increases, the retarding force offered to it by the fluid also increases.

We can study its consequences on an object falling through a viscous medium.

- Is Stokes' law applicable only to spherical bodies?

- In the light of Stokes' law, can the retardation of the body moving in the liquid be seen due to adhesive force between the molecules of body and fluid; and cohesive force between the molecules of fluid itself?

12. TERMINAL VELOCITY



Imagine you are about to jump out of an airplane. At the last minute, you have to decide whether to take a parachute or not. What would you choose? Obviously, you would choose the parachute! Although we all know that it is better to jump from a plane with a parachute than without it, have you ever thought about why?

As the object falls, the force of gravity initially causes it to continuously speed up as predicted by Isaac Newton.

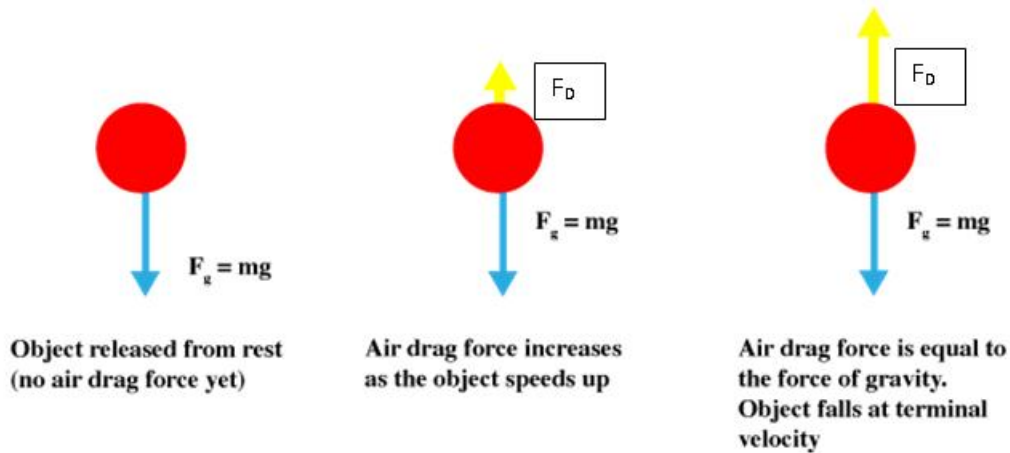
As it gets faster and faster, the air drag force increases as per Stokes law ($F \propto v$) until eventually, the air drag force is exactly equal to the force of gravity, and there is no net force acting on the object.

If these two forces are exactly balanced, the object will no longer speed up or slow down but will continue falling at a constant velocity, called the **terminal velocity**.

Since the air drag force depends heavily on the size and shape of the object, objects with a large surface area (like a parachute) will have a much lower terminal velocity than objects with a smaller surface area (like a person falling from a plane).

The weight of the object does affect the air drag force on hit and, therefore, its terminal velocity.

However, it is not the most important factor. This explains



Why a flat piece of paper will fall more slowly as compared to the same paper after it has been crumpled into a ball?



<https://www.google.com/search?site=imghp&tbm=isch&q=crumpled%20paper%20ball&tbs=sur:fmc#imgrc=t2IOTa07UpUHIM:>

The paper weighs the same, but the air drag forces on the ball have decreased because its surface area has decreased.

This causes the crumpled paper to have a higher terminal velocity than the flat paper.

This also explains why a parachute can lower your terminal velocity when you jump from an airplane. The parachute has a very large surface area and a large drag force and a relatively small mass, so it experiences much higher air drag forces than you.

CONSIDER THESE

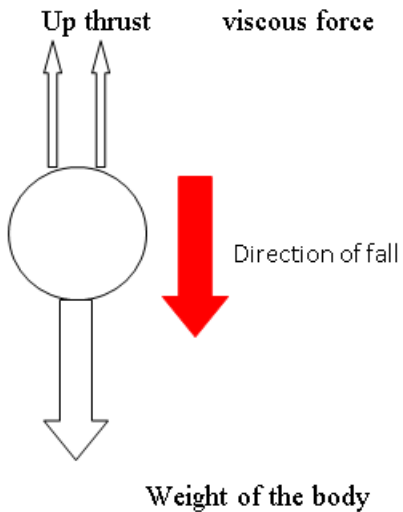
- If a rain drop is falling from a height of 7km towards the surface of earth before colliding with the surface of earth up to what distance will it maintain its terminal velocity?
- If two very small bodies of same mass and size are made to fall from the same height in two different regions of the earth, one body is falling in area where

the atmosphere is hot due to extreme summer and other where atmosphere is cold due to winter, considering the viscous force which body fall strike to the surface of earth earlier?

- An airplane seems to fly with constant speed, does it also move with the terminal velocity?

13. EXPRESSION FOR TERMINAL VELOCITY

Consider a spherical body of radius 'a' falling through a viscous liquid of density ρ and Coefficient of viscosity η
Let σ be density of spherical body.



As the body falls various forces (3 in number) acting on it are

1. **weight** of the body acting vertically downwards

$$W = mg = \frac{4}{3} \pi a^3 \sigma g$$

2. **upward force** = weight of the fluid displaced

$$U = mg = \frac{4}{3} \pi a^3 \rho g$$

3. **viscous force** = $6 \pi \eta r v$

It accelerates initially due to gravity. As the velocity increases, the retarding force also increases.

Finally

When viscous force plus up thrust, together become equal to force due to gravity, the net force becomes zero and so does the acceleration.

The spherical body then descends with a constant velocity called terminal velocity v_t .

When the spherical body attains terminal velocity, then

Weight of body = up thrust + viscous force

$W = U + F$

$$\frac{4}{3} \pi r^3 \sigma g = \frac{4}{3} \pi r^3 \rho g + 6 \pi \eta r v$$

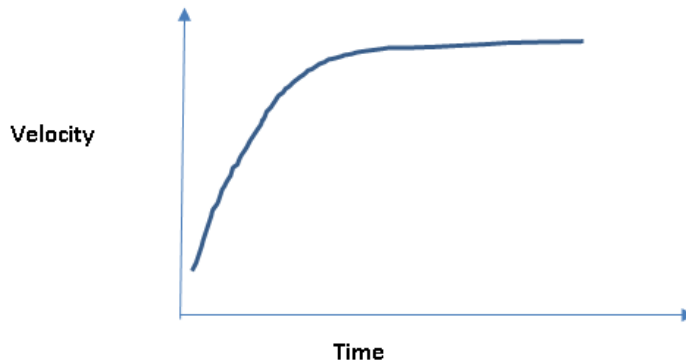
$$v = \frac{\frac{4}{3} \pi r^3 (\sigma - \rho) g}{6 \pi \eta r} = \frac{2 r^2 (\sigma - \rho) g}{9 \eta}$$

v is constant as the net force on the body is zero.

- If the body falls in vacuum the velocity would continuously increase
 $v^2 = 2g h$
- In case there was no viscosity the net force on the falling body would be $W - \text{up thrust}$
- **In case the fluid viscosity is relevant the viscous drag is depended on instantaneous speed of the falling body. So the net force would decrease and tend to become zero.**

According to Newton's first law "a body continues in its state of uniform motion unless and until an external unbalanced force acts on the body".

The graph shows how the velocity of a falling object in a viscous fluid first increases and then becomes constant. The constant velocity is called terminal velocity



So , the terminal velocity v_t is

1. **Directly proportional to the square of the radius of the sphere. That is why bigger rain drops fall with a greater velocity compared to the smaller rain drops and hurt more.**
2. **Inversely proportional to the viscosity of the medium. The more viscous the fluid, the smaller the terminal velocity attained by a body.**
3. **Directly proportional to the difference in the density of the body and the fluid.**

a) If $\rho_b > \rho_l$, the body will attain terminal velocity in downward direction.



b) If $\rho_b < \rho_l$, the body will attain terminal velocity in upward direction. That is why air bubbles in a liquids are seen to move in upward direction.



<https://openphoto.net/volumes/sizes/danieljaeger/9684/2.jpg>

c) If $\rho_b = \rho_l$, the body remains suspended in the fluid.

If a large amount of suspended solid particles are present in water, it will appear turbid in appearance. The turbidity depends upon fineness and concentration of particles present in water.

EXAMPLE

The terminal velocity of a copper ball of radius 2.0 mm falling through a tank of oil at 20°C is 6.5 cm s⁻¹.

Compute the viscosity of the oil at 20°C.

Density of oil is $1.5 \times 10^3 \text{ kg m}^{-3}$, density of copper is $8.9 \times 10^3 \text{ kg m}^{-3}$

SOLUTION

We have

$$v_t = 6.5 \times 10^{-2} \text{ ms}^{-1},$$

$$a = 2 \times 10^{-3} \text{ m},$$

$$g = 9.8 \text{ ms}^{-2},$$

$$\rho_b = 8.9 \times 10^3 \text{ kg m}^{-3},$$

$$\rho_l = 1.5 \times 10^3 \text{ kg m}^{-3}.$$

Using

$$v = \frac{\frac{4}{3}\pi r^3(\sigma - \rho)g}{6\pi\eta r} = \frac{2r^2(\sigma - \rho)g}{9\eta}$$

$$\eta = \frac{2r^2(\sigma - \rho)g}{9v}$$

$$\eta = \frac{2 \times (2 \times 10^{-3})^2 \text{ m}^2 \times 9.8 \text{ ms}^{-1} \times 7.4 \times 10^3 \text{ kg m}^{-3}}{9 \times 6.5 \times 10^{-2} \text{ ms}^{-1}}$$

$$= 9.9 \times 10^{-1} \text{ kg m}^{-1} \text{ s}^{-1}$$

EXAMPLE

A drop of water of radius 0.0015mm is falling in air. If the coefficient of viscosity of air is $1.8 \times 10^{-5} \text{ kg/(m-s)}$, what will be the terminal velocity of the drop?

(Density of water = $1.0 \times 10^3 \text{ kg/m}^3$ and $g = 9.8 \text{ N kg}^{-1}$).

Density of air can be neglected.

SOLUTION-

By Stokes' law, the terminal velocity of a water drop of radius r is given by

$$v = \frac{2r^2(\rho - \sigma)g}{9\eta}$$

Where ρ is the density of water, σ is the density of air and η the coefficient of viscosity of air. Here σ is negligible and $r = 0.0015 \text{ mm} = 1.5 \times 10^{-3} \text{ mm} = 1.5 \times 10^{-6} \text{ m}$. Substituting the values:

$$v = \frac{2}{9} \times \frac{(1.5 \times 10^{-6})^2 \times (1.0 \times 10^3) \times 9.8}{1.8 \times 10^{-5}}$$

$$v = 2.72 \times 10^{-4} \text{ m/s}$$

THINK ABOUT THESE

- Does the property of viscosity increases with the increase in the roughness of surface through which the fluid is flowing?
- Is viscous force a kind of inertia force?
- Why the direction of viscous force is against the flow?

- If we compare force of viscosity with friction force, then friction force does not solely depend of the object or the surface on which the object is moving? Then why viscosity, depending solely on the fluid is acting as a force of friction?
- What are those internal factors due to which the value of viscosity differs for different fluids?
- As we heat a fluid it becomes less viscous explain on the basis of change in density?
- The direction of viscous drag is opposite to the direction of motion of the ball, what is the direction of viscous drag when a ball is thrown up in the air?

14. DETERMINE THE COEFFICIENT OF VISCOSITY OF A GIVEN VISCOUS LIQUID BY MEASURING TERMINAL VELOCITY OF A GIVEN SPHERICAL BODY IN THE LABORATORY

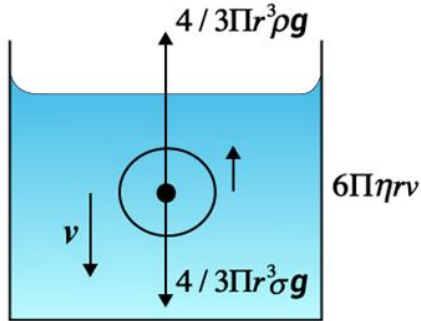
APPARATUS AND MATERIAL REQUIRED

- A wide bore tube of transparent glass/acrylic (approximately 1.25 m long and 4 cm diameter),
- lead balls of known diameters between 1.0 mm to 3 mm,
- transparent viscous liquid (castor oil/glycerin),
- Two stop watches
- laboratory stand,
- forceps,
- rubber bands,
- a thermometer (0-50 °C), and
- metre scale.

PRINCIPLE

When a spherical body of radius r and density σ falls freely through a viscous liquid of density ρ and viscosity η , with terminal velocity v , then the sum of the upward buoyant force and viscous drag, force F , is balanced by the downward force

weight of the ball. = Buoyant force on the ball + viscous force



$$\frac{4}{3}\pi r^3 \sigma g = \frac{4}{3}\pi r^3 \rho g + 6\pi \eta r v$$

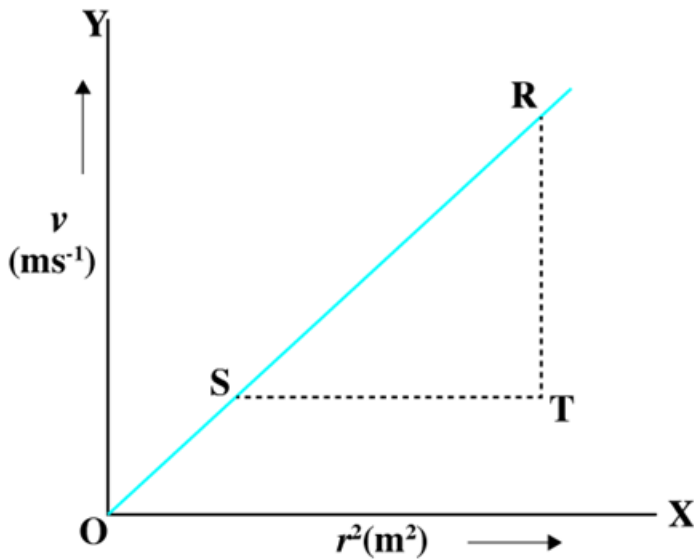
$$v = \frac{\frac{4}{3}\pi r^3 (\sigma - \rho) g}{6\pi \eta r} = \frac{2r^2 (\sigma - \rho) g}{9\eta}$$

$$\eta = \frac{2r^2 (\sigma - \rho) g}{9v}$$

Where v is the terminal velocity, the constant velocity acquired by a body while moving through viscous fluid under application of constant force.

The terminal velocity depends directly on the square of the size (diameter) of the spherical ball.

Therefore, if several spherical balls of different radii are made to fall freely through the viscous liquid then a plot of v vs r^2 would be a straight line



The shape of this line will give an average value of v/r^2 which may be used to find the coefficient of viscosity η of the given liquid.

Thus

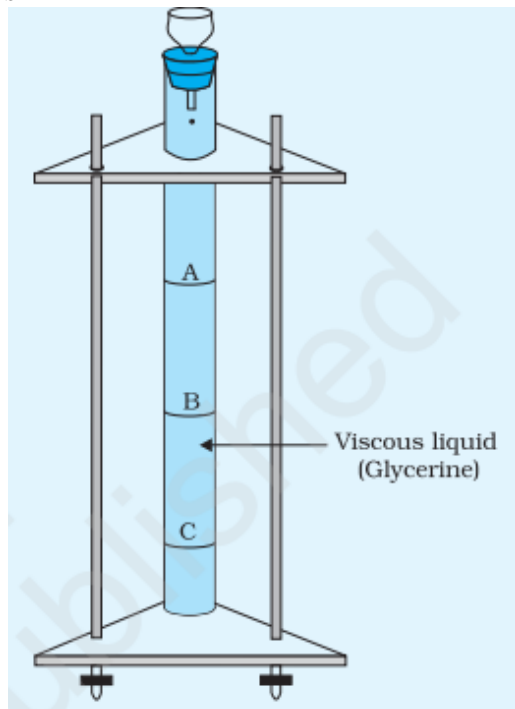
$$\eta = \frac{2r^2(\sigma - \rho)g}{9v}$$

Will give us the viscosity of the liquid in the glass tube (our experimental liquid)

The relation holds good if the liquid through which the spherical body falls freely is in a cylindrical vessel of radius $R \gg r$ and the height of the cylinder is sufficient enough to let the ball attain terminal velocity. At the same time the ball should not come in contact with the walls of the tube

PROCEDURE

- i. **Find the least count of the screw gauge,**
- ii. Find the least count of the thermometer and note the room temperature
- iii. **Take a wide bore tube carefully fill it with the given transparent viscous liquid (say glycerine).**
- iv. Fix the tube vertically in the clamp stand
- v. **Ensure that there is no air bubble inside the viscous liquid in the wide bore tube.**
- vi. Put three rubber bands A, B, and C around the wide bore tube dividing it into four portions



such that $AB = BC$, each about 25 cm. The rubber band A should be around 30 cm below the mouth of the wide bore tube (length sufficient to allow the ball to attain terminal velocity).

- vii. **Separate a set of clean and dry lead balls of different radii. The set should include four or five identical lead balls of same radii (r_1).**
- viii. Rinse these balls thoroughly with the experimental viscous liquid (glycerin) in a petri dish or a watch glass.
This is done so that air bubbles do not form on the lead balls as it goes down in the glycerin
- ix. **With the help of forceps hold one of the balls of radius r_1 near the top of tube. Allow the ball to fall freely. The ball, after passing through the inlet tube, will fall along the axis of the liquid column.**



Watch video to see the method to determine the diameter of the lead shot using a screw gauge

- x. **Take two stop watches and start both of them simultaneously** as the spherical ball passes through the rubber band A. Stop one of the watches as the ball passes through the band B. Allow the second stop-watch to continue and stop it when the ball crosses the band C, **or you may carefully record the time of fall from A to B by one and from B to C by the other stop watch.**



- xii. Note the times t_1 and t_2 as indicated by the two stop watches, t_1 is then the time taken by the falling ball to travel from A to B and t_2 is the time taken by it in falling from A to C. If terminal velocity had been attained before the ball crosses A, If it is not so, repeat the experiment with steel ball of same radii after adjusting the positions of rubber bands.
- xiii. Repeat the experiment for other balls of different diameters.
- xiv. Obtain terminal velocity for each ball.
- xv. Plot a graph between terminal velocity, v and square of the radius of spherical ball, r^2 . It should be a straight line.
- xvi. Find the slope of the line and hence determine the coefficient of viscosity of the liquid using the relation

OBSERVATION TABLE

1. Least count of thermometer = °C.
2. Temperature of experimental liquid (glycerin) $\theta = \dots$ °C.
3. Least count of stop-watch = ...
4. Least count of screw gauge = ...
5. Density of material of balls (lead) $\sigma = \dots$ kg m⁻³
6. Density of experimental viscous liquid(glycerine) $\rho = \dots$ kg m⁻³
7. Internal diameter of the wide bore tube =... cm = ... m
8. Length of wide bore tube = ... cm = ... m
9. Distance between A and B = .25.. cm = ... m
10. Distance between B and C = 25 ... cm = ... m
11. Acceleration due to gravity at the place of experiment, = 9.8 ms⁻²

Table to record radius and time of fall

S. No.	Diameter of spherical balls		Square of the radius of the balls r^2 (m ²)	Time taken for covering distance $h = \dots$ cm between rubber bands				Terminal Velocity $v = \frac{h}{t}$ (m ⁻¹)
	d (cm)	$r = d/2$ (m)		A and B t_1 (s)	A and C t_2 (s)	B and C $t_3 = t_2 - t_1$ (s)	Mean time $t = \frac{t_1 + t_3}{2}$ (s)	
1								
2								
3								

Plot a graph. Find its slope and calculate the coefficient of viscosity report your result with proper units

PRECAUTIONS AND SOURCES OF ERROR

- In order to minimize the effects, although small, on the value of terminal velocity (more precisely on the value of viscous drag, force F), the radius of the wide bore tube containing the experimental viscous liquid should be much larger than the radius of the falling spherical balls.
- The steel balls should fall without touching the sides of the tube.
- The ball should be dropped gently in the tube containing viscous/ liquid.

DISCUSSION

- Ensure that the ball is spherical. Otherwise formula used for terminal velocity will not be valid.
- Motion of falling ball must be translational.
- Diameter of the wide bore tube should be much larger than that of the spherical ball.

THINK ABOUT THIS

- **Do all the raindrops strike the ground with the same velocity irrespective of their size?**
- **Is Stokes' law applicable to body of shapes other than spherical?**
- **What is the effect of temperature on coefficient of viscosity of the liquid?**
- **Value of η can be calculated for steel balls of different radii and compared with that obtained from the experiment.**

- To find viscosity of mustard oil [Hint: Set up the apparatus and use mustard oil instead of glycerin in the wide bore tube].
- To check purity of milk [Hint: Use mustard oil in the tall tube. Take an eye dropper, fill milk in it. Drop one drop of milk in the oil at the top of the wide bore tube and find its terminal velocity. Use the knowledge of coefficient of viscosity of mustard oil to calculate the density of milk].
- Study the effect of viscosity of water on the time of rise of air bubble [Hint: Use the bubble maker used in an aquarium. Place it in the wide bore tube. Find the terminal velocity of rising air bubble]

15. SUMMARY

The property of a fluid by virtue of which an internal frictional force acts between its different layers which opposes their relative motion is called viscosity.

- The ratio of the shear stress to the time rate of shearing strain is known as coefficient of viscosity, η .
- As the temperature rises the coefficient of viscosity, η falls. In a gas the temperature rise increase the random motion of atoms and viscosity increases
- Stokes' law states that the viscous drag force F on a sphere of radius a moving with velocity v through a fluid of viscosity is, $F = -6 \pi \eta a v$.
- When an object which is falling under the influence of gravity or subject to some other constant driving force is subject to a resistance or drag force which increases with velocity, it will ultimately reach a maximum velocity where the drag force equals the driving force. This final constant velocity of motion is called a "terminal velocity".